PHYS 2601 (Fall 2025): Problem Set 3
Due date: September 28, 9am. 50% penalty on late homework.

From Vibrations and Waves (King)
Problem 2.1 (5 pts)
Problem 2.6 (5 pts)
Problem 2.7 (5 pts)

From Vibrations and Waves (French)
Problem 3.15 (5 pts)

3–15 Many oscillatory systems, although the loss or dissipation mechanism is not analogous to viscous damping, show an exponential decrease in their stored *average* energy with time, $\overline{E} = \overline{E}_0 e^{-\gamma t}$. A Q for such oscillators may be defined using the definition $Q = \omega_0/\gamma$, where ω_0 is the natural angular frequency.

(a) When the note "middle C" on the piano is struck, its energy of oscillation decreases to one half its initial value in about 1 sec. The frequency of middle C is 256 Hz. What is the Q of the system?

(b) If the note an octave higher (512 Hz) takes about the same time for its energy to decay, what is its Q?

(c) A free, damped harmonic oscillator, consisting of a mass $m = 0.1$ kg moving in a viscous liquid of damping coefficient b ($F_{\text{viscous}} = -bv$), and attached to a spring of spring constant $k = 0.9$ N/m, is observed as it performs damped oscillatory motion. Its average energy decays to $1/e$ of its initial value in 4 sec. What is the Q of the oscillator? What is the value of b?

Problem 3.16 (10 pts)

3–16 According to classical electromagnetic theory an accelerated electron radiates energy at the rate Ke^2a^2/c^3, where $K = 6 \times 10^9$ N$-$m^2/C^2, e = electronic charge (C), a = instantaneous acceleration (m/sec^2), and c = speed of light (m/sec).

(a) If an electron were oscillating along a straight line with frequency ν (Hz) and amplitude A, how much energy would it radiate away during 1 cycle? (Assume that the motion is described adequately by $x = A \sin 2\pi\nu t$ during any one cycle.)

(b) What is the Q of this oscillator?

(c) How many periods of oscillation would elapse before the energy of the motion was down to half the initial value?

(d) Putting for ν a typical optical frequency (i.e., for visible light) estimate numerically the approximate Q and "half-life" of the radiating system.

Extra credit problems:

Extra credit problem 1 – Numerical solution of the simple pendulum (5 points)

We have discussed in class that the simple pendulum cannot be solved with analytical methods. See section 1.3.1. To find solutions, we approximated in the equation of motion $\sin\theta \approx \theta$, which is only valid for small displacement angles. The purpose of this problem is to numerically solve the equation of motion for a simple pendulum *without* the small angle approximation.

(a) The book by King discusses the concept how to numerically solve the equation of motion for an undamped simple pendulum by considering the change of θ and $\dot{\theta}$ in discrete time steps δt. Read section 1.3.4 in the book carefully. The numerical method described is called *Runge-Kutta method* and you will find more information about it online.

(b) Following the method, numerically determine the displacement θ of the pendulum as a function of time based on the differential equation

$$\ddot{\theta}(t) = -\frac{g}{l}\sin\theta(t). \tag{1}$$

To implement the numerical approach you can use a software of your choice, e.g., Excel, Python, or Mathematica. For the length of the pendulum use $l = g/\pi^2$, where g is the gravitational acceleration of the Earth. **Provide a description and a screen shot of your code.**

(c) Determine the displacement as a function of time for the following parameters: (i) $\theta(t = 0) = 0.1$, $\dot{\theta}(t = 0) = 0$, $\delta t = 0.02$ s; (ii) $\theta(t = 0) = 1$, $\dot{\theta}(t = 0) = 0$, $\delta t = 0.02$ s. Plot the displacement as a function of time up to 20 s, i.e. perform 1000 time steps in the simulation. **Provide a table with the first few time steps listing the values of time t, $\theta(t)$, and $\dot{\theta}(t)$ for both cases.**

(d) Compare the results of (c) with the function $\theta(t) = \theta(t = 0)\cos\omega_0 t$, where $\omega_0 = \sqrt{g/l}$. Comment on the similarities and differences.

Extra credit problem 2 – Numerical solution of the damped simple pendulum (5 points)

Using the numerical method, it is straightforward to take into account the effects of damping. Extend your approach developed in Problem 1 to solve the equation of motion in the presence of damping, given by the differential equation

$$\ddot{\theta}(t) = -\frac{g}{l}\sin\theta(t) - \frac{b}{m}\dot{\theta}(t). \tag{2}$$

Provide a description and a screen shot of your code. For the length of the pendulum again use $l = g/\pi^2$, for the damping constant $b = 0.1$, and for the mass $m = 1$.

(a) Determine the displacement as a function of time for the following parameters: (i) $\theta(t=0) = 10°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$ s; (ii) $\theta(t=0) = 90°$, $\dot{\theta}(t=0) = 0$, $\delta t = 0.02$ s. Plot the displacement as a function of time up to 20 s. Feel free to experiment with other starting parameters and observe how the system behaves. **Provide a table with the first few time steps listing the values of time t, $\theta(t)$, and $\dot{\theta}(t)$ for both cases.**

(b) Compare the results of (a) with the functions $\theta(t) = \theta(t=0)\cos\omega_0 t$ (with $\omega_0 = \sqrt{g/l}$) and $\theta(t) = \theta(t=0)e^{-\gamma t/2}\cos\omega t$ with $\omega = \sqrt{\omega_0^2 + \gamma^2/4}$ and $\gamma = b/m$. Comment on similarities and differences.

In your solutions, please provide written comments (in addition to the math) that show your reasoning to receive full credit.

Please submit solutions electronically as a pdf document to **gradescope** (handwritten or typeset).